Description and Solution of an Unreported Intrinsic Bias in Photon Mapping Density Estimation with Constant Kernel

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Photon Mapping

- Developed in 1995 by Jensen
- Very successful algorithm for complex illumination

Phases

- Photon Tracing
- Ray tracing (from the eye)
- Density Estimation (photon map query)

Biases

Previously known biases:

- Proximity Bias: The algorithm converges to the weighted average irradiance in a neighbourhood
- Boundary Bias: There are no impacts outside borders
- Topological Bias: Assumption of locally planar surface

New bias source:

- Overestimation bias:

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Probability distribution

- An experiment or algorithm may have a (pseudo-) random outcome.
- A probabilistic distribution function f gives the probability mass of each possible outcome
- An integral of f over a (non-zero-measure) set of outcomes gives the probability that one of the outcomes happens in a realization of the experiment.

Order Statistics

- We may sort the results after repeated trials, to calculate probabilities of minima and maxima
- These probabilities depend on the f and the number of experiments
- In general, the i^{th} order statistic is the value of the i^{th} position of the sorted vector of results, with probability distribution $f_{X(i)}$

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Modelling

- The distribution of impacts f follows the irradiance function I
- We sort the n impacts according to the distance to P
- We model the photon map query of the k^{th} nearest impact using order statistics

To simplify the study, we use a unit disc with uniform radiance

- Flux of a photon: $\phi = \pi \ I(P)/n$
- Estimator: $\widehat{I}(r_k) = \frac{k\phi}{\pi r_k^2} = \frac{k \ I(P)}{n r_k^2}$

Results of the study

- Expected value:

$$E[\widehat{I}(r_k)] = \int_0^1 \widehat{I}(r_k) f_{X_{(k)}}(r_k) \mathrm{d}r_k = \frac{k}{k-1} I(P)$$

- *Overestimation!*
- Fix: Take the distance of the k^{th} nearest impact, but the flux of the $k-1$ nearest impacts

$$E[\widehat{I_{k-1}}(r_k)] = \int_0^1 \widehat{I_{k-1}}(r_k) f_{X_{(k)}}(r_k) \mathrm{d}r_k = \frac{k-1}{k-1} I(P) = I(P)$$

Modelling

- Unit sphere with uniform power density PD, total power W
- The distribution of photons follows the power density
- Use order statistics similarly to the 2D case

$$PD = 3W/4\pi \quad ; \quad \phi = W/n$$

- Estimator:

$$\widehat{PD}(r_k) = \frac{k\phi}{\frac{4}{3}\pi r_k^3}$$

Results of the study

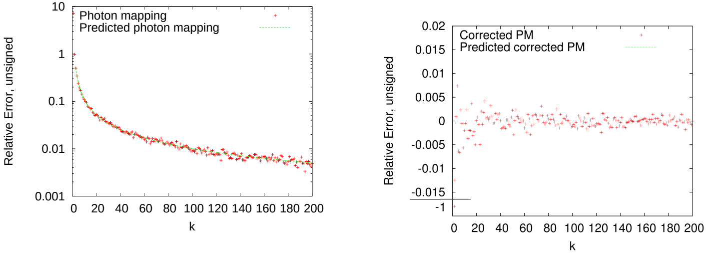
- Expected value:

$$E[\widehat{PD}(r_k)] = \int_0^1 \widehat{PD}(r_k) f_{X_{(k)}}(r_k) \mathrm{d}r_k = \frac{k}{k-1} PD$$

- *Overestimation again!*
- Fix: Take the distance of the k^{th} nearest impact, but the power of the $k-1$ nearest impacts

$$E[\widehat{PD_{k-1}}(r_k)] = \int_0^1 \widehat{PD_{k-1}}(r_k) f_{X_{(k)}}(r_k) \mathrm{d}r_k = \frac{k-1}{k-1} PD = PD$$

Empirical study of uniform lighting



Relative error of original photon mapping and our corrected photon mapping for a uniform distribution of photons, as a function of k; theoretical prediction of the error

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Conclusions

- We have seen that a mathematical modelling of Photon Mapping can be used to understand the algorithm better.
- A new bias source (overestimation) has been identified and eliminated
- The study has been validated by empirical studies

Future work

- Study filtering kernels (article under review)
- Study stratified sampling
- Apply the framework to other Photon Map variants
- We encourage other researchers to use and extend the framework

Questions and Comments

Thank you for your attention.

Questions and comments are welcome.