Description and Solution of an Unreported Intrinsic Bias in Photon Mapping Density Estimation with Constant Kernel

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2 Order Statistics
   - Preliminaries
   - Definition

3 Overestimation Bias
   - Classic photon mapping
   - Volumetric effects
   - Empirical Study

4 Conclusions
Photon Mapping

- Developed in 1995 by Jensen
- Very successful algorithm for complex illumination
Phases

- Photon Tracing
- Ray tracing (from the eye)
- Density Estimation (photon map query)
Previously known biases:

- **Proximity Bias**: The algorithm converges to the weighted average irradiance in a neighbourhood
- **Boundary Bias**: There are no impacts outside borders
- **Topological Bias**: Assumption of locally planar surface

New bias source:

- **Overestimation bias:**
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Probability distribution

- An experiment or algorithm may have a (pseudo-) random outcome.
- A probabilistic distribution function $f$ gives the probability mass of each possible outcome.
- An integral of $f$ over a (non-zero-measure) set of outcomes gives the probability that one of the outcomes happens in a realization of the experiment.
Order Statistics

- We may sort the results after repeated trials, to calculate probabilities of minima and maxima.
- These probabilities depend on the $f$ and the number of experiments.
- In general, the $i^{th}$ order statistic is the value of the $i^{th}$ position of the sorted vector of results, with probability distribution $f_{X(i)}$. 
Modelling

- The distribution of impacts $f$ follows the irradiance function $I$
- We sort the $n$ impacts according to the distance to $P$
- We model the photon map query of the $k^{th}$ nearest impact using order statistics

To simplify the study, we use a unit disc with uniform radiance

- Flux of a photon: $\phi = \pi \frac{I(P)}{n}$
- Estimator: $\hat{I}(r_k) = \frac{k\phi}{\pi r_k^2} = \frac{kI(P)}{nr_k^2}$
Results of the study

- Expected value:
  \[
  E[\hat{I}(r_k)] = \int_0^1 \hat{I}(r_k) f_{X(k)}(r_k) dr_k = \frac{k}{k - 1} I(P)
  \]

- Overestimation!

- Fix: Take the distance of the \(k^{th}\) nearest impact, but the flux of the \(k - 1\) nearest impacts
  \[
  E[\hat{I}_{k-1}(r_k)] = \int_0^1 \hat{I}_{k-1}(r_k) f_{X(k)}(r_k) dr_k = \frac{k - 1}{k - 1} I(P) = I(P)
  \]
Unit sphere with uniform power density $PD$, total power $W$

The distribution of photons follows the power density

Use order statistics similarly to the 2D case

\[ PD = \frac{3W}{4\pi} ; \quad \phi = \frac{W}{n} \]

Estimator:

\[ \hat{PD}(r_k) = \frac{k\phi}{\frac{4}{3}\pi r_k^3} \]
Results of the study

- Expected value:
  \[
  E[\hat{PD}(r_k)] = \int_0^1 \hat{PD}(r_k)f_X(k)(r_k)dr_k = \frac{k}{k-1}PD
  \]

- Overestimation again!

- Fix: Take the distance of the \(k^{th}\) nearest impact, but the power of the \(k-1\) nearest impacts
  \[
  E[\hat{PD}_{k-1}(r_k)] = \int_0^1 \hat{PD}_{k-1}(r_k)f_X(k)(r_k)dr_k = \frac{k-1}{k-1}PD = PD
  \]
Empirical study of uniform lighting

Relative error of original photon mapping and our corrected photon mapping for a uniform distribution of photons, as a function of $k$; theoretical prediction of the error
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- We have seen that a mathematical modelling of Photon Mapping can be used to understand the algorithm better.
- A new bias source (overestimation) has been identified and eliminated
- The study has been validated by empirical studies
Future work

- Study filtering kernels (article under review)
- Study stratified sampling
- Apply the framework to other Photon Map variants
- We encourage other researchers to use and extend the framework
Questions and Comments

Thank you for your attention.

Questions and comments are welcome.